Linear Mixed-Effect Models

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Abstract: *Linear Mixed-Effect Models* are an extension of *Linear Regression* that describe the relationship between response variable $\mathcal{Y}$ and independent variables X such that the coefficients can vary with respect to one or more grouping variables and hence at least one independent covariate should be categorical.
Mixed-Effects models find use in *longitudinal* or repeated measures study, where repeated measurements are made on *experimental* or *observational* units.
Mixed-Effects models make use of constrained optimisation to arrive at the Maximum Likelihood or Restricted Maximum Likelihood estimate of the parameters.

1. PROBLEM DESCRIPTION

Consider a Linear Regression Problem:

$$\mathcal{Y} = X\beta + \epsilon$$

Where $\epsilon \sim \mathcal{N}(0, \sigma^2 I)$ and β is a p-dimensional coefficient vector; X is $n \times p$ model matrix. There are two parameters in this model: β and σ^2

and hence for a linear model:

$$y \sim \mathcal{N}(X\beta, \sigma^2 I)$$

Mixed-effects models the response with an additional "random-effect" $\mathcal{B}$ such that:

$$(\mathcal{Y}|\mathcal{B} = b) = \mathcal{N}(XB + Zb, \sigma^2 I)$$

where Z is a $n \times q$ model matrix just like X but for the random-effect covariates $\mathcal{B}$ which we fix at b and then model b as another normal random variable:

$$\mathcal{B} \sim \mathcal{N}(0, \Sigma)$$

where Σ is a parameterized $q \times q$ covariance matrix. The parameter estimation now can be down by separating(profiling) the log-likelihood.(Details Skipped, since I do not understand them yet)

2. GOALS

- Understand the derivation/math behind parameter estimation
- Use available modeling libraries to demonstrate at least one use case of mixed-effects models

3. REFERENCES

- Pinherio, J. C., and D. M. Bates. Mixed-Effects Models in S and S-PLUS. Statistics and Computing Series, Springer, 2004.
- Bates, Douglas, et al. "Fitting linear mixed-effects models using lme4." arXiv preprint arXiv:1406.5823 (2014).