Homework on Overlap and Nonoverlap Pattern Count(due Oct. 08, 2015)

Homework 1: Let $A_1A_2\cdots A_n\cdots$ be IID sequences taking values 0 or 1 with $P(A_i = 1) = p$ and $P(A_i = 0) = q = 1 - p$. Let $U = 11$ and $V = 101$. Let $N_W(n)$ be the number of occurrences of word W in $A_1A_2\cdots A_n$. From the first principle without using the formulas in the lecture,

- Calculate $E(N_U(n))$ and $E(N_V(n))$.
- Calculate $Var(N_U(n))$, $Var(N_V(n))$, and $Cov(N_U(n), N_V(n))$.

Homework 2 (Continuation of problem 1 from last week). Under the same assumptions as in problem 1,

- Calculate the overlap bits $\beta_{UU}(j)$, $\beta_{VV}(j)$, $\beta_{UV}(j)$, and $\beta_{VU}(j)$.
- Calculate the variance and covariance again of $N_U(n)$ and $N_V(n)$ using Corollary 12.1.
- Calculate the appropriate variance of $\sqrt{n}\left(N_V(n)/n - \hat{p_n}^2(1-\hat{p_n})\right)$, where $p_n = N_1(n)/n$ where $N_1(n)$ is the number of 1's in the first n positions.

Homework 3. Same settings as in Problem 1. Let $N_{00}(n)$ be the number of "00"s in the sequence and $N_0(n)$ be the number of "0"s in the sequence.

1. Calculate the approximate variance of

$$\lim_{n\to\infty} \sqrt{n} \begin{pmatrix} \frac{N_{00}(n)}{n} - q^2 \\ \frac{N_0(n)}{n} - q \end{pmatrix}$$

2. Using delta method to calculate the approximate variance of $\sqrt{n}\left(\frac{N_{00}}{n} - \left(\frac{N_0(n)}{n}\right)^2\right)$

Homework 4. Consider IID sequence of letters R and Y with $p(R) = p$. Let $\mathbf{w} = RYR$. Let $T_1(\mathbf{w})$ be the number of letters to see the first renewal of pattern $\mathbf{w}$, $T_{\mathbf{w}}^{(r)}$ be the number of letters to see the r-th renewal and $M_{\mathbf{w}}(n)$ be the number of non-overlapping occurrences of word $\mathbf{w}$ within the first n letters.

1. Calculate $E(T_1(\mathbf{w}))$ and $\text{var}(T_1(\mathbf{w}))$.
2. What is the limiting distribution of $T_{\mathbf{w}}^{(r)}$ as r tends to infinity?
3. What is the limiting distribution of $M_{\mathbf{w}}(n)$ as n tends to infinity?