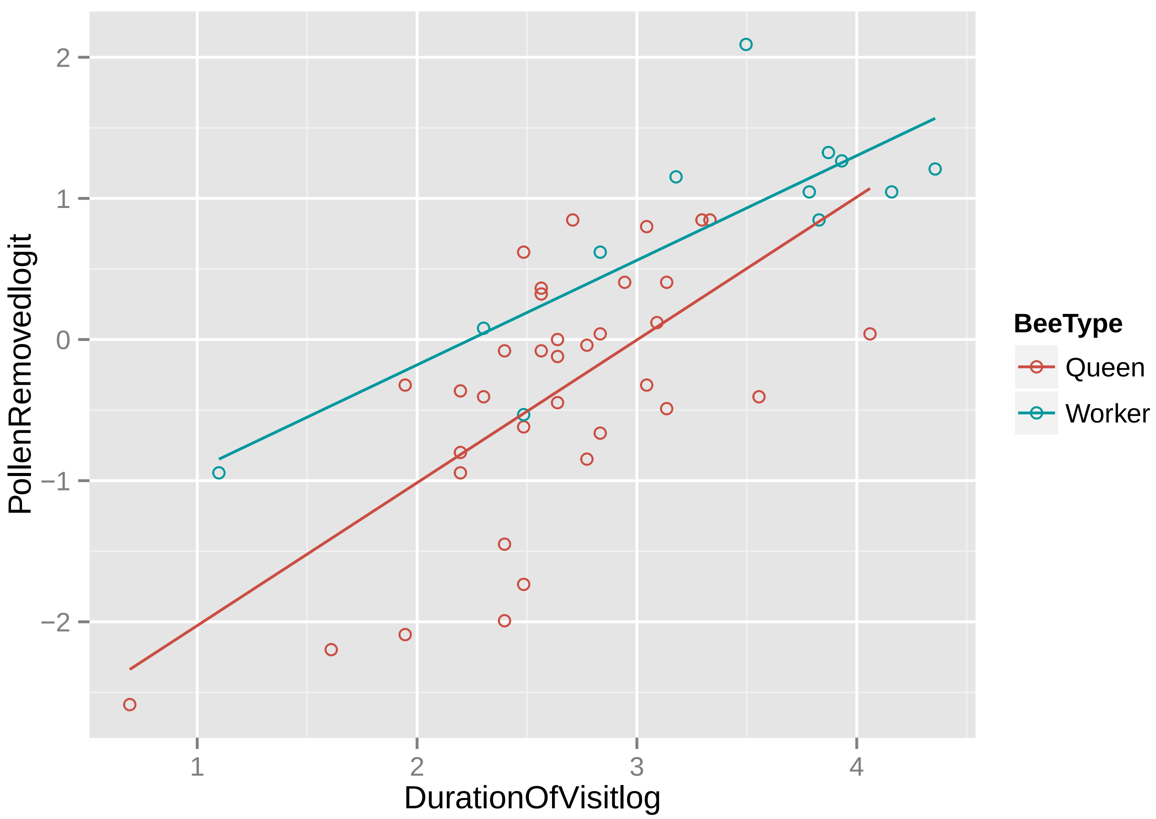
MATH-650 Assignment 8

Saket Choudhary (USCID: 2170058637) (skchoudh@usc.edu)

09/28/2015

Chapter 11: 10

```
library(ggplot2)
data <- read.csv('data_ch9_16.csv', header=T)
data$PollenRemovedlogit = log(data$PollenRemoved/(1-data$PollenRemoved))
data$DurationOfVisitlog = log(data$DurationOfVisit)
ggplot(data, aes(x=DurationOfVisitlog, y=PollenRemovedlogit, color=BeeType)) +
    geom_point(shape=1) +
    scale_colour_hue(l=50) +
    geom_smooth(method=lm,
                se=FALSE)
```



$$\mu\{PollenRemovedLogit|DuratioOfVisitlog, BeeType\} = \beta_0 + \beta_1 DurationOfVisitlog \\ + \beta_2 BeeType + \beta_3 BeeType * DurationOfVisitlog$$

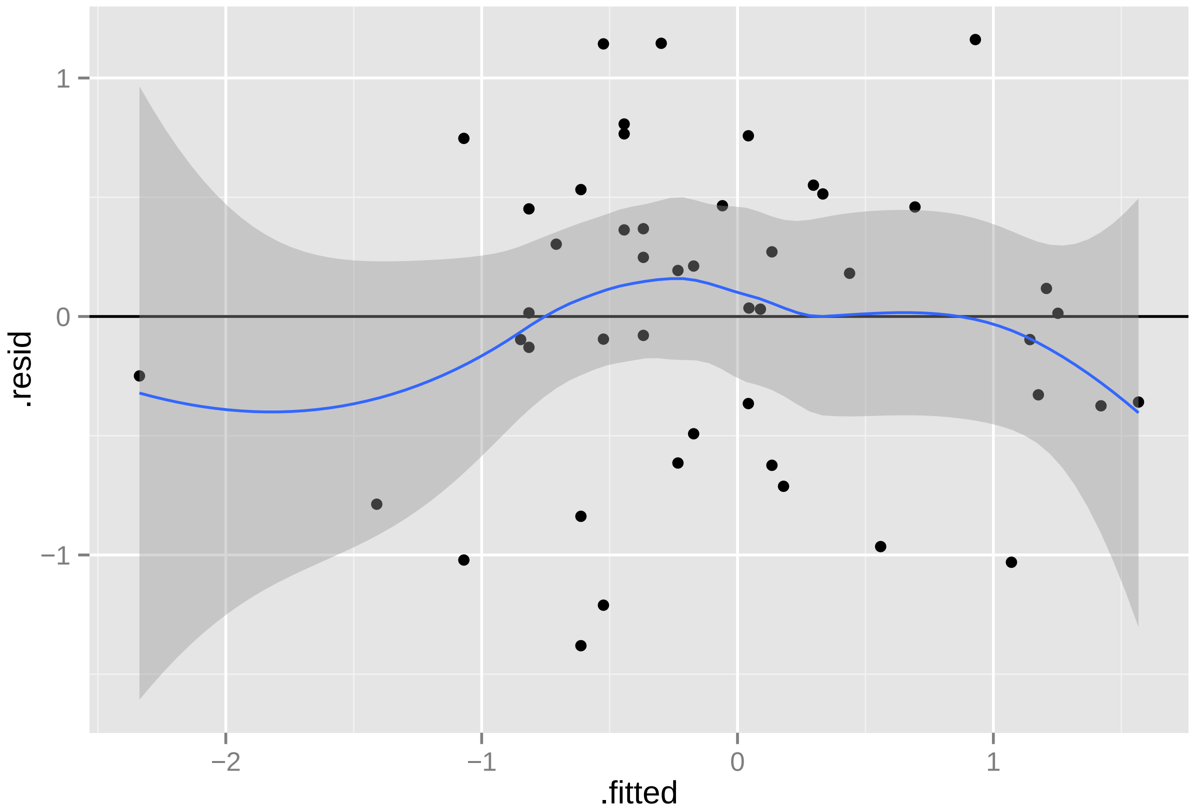
```
lmfit <- lm(PollenRemovedlogit ~ BeeType + DurationOfVisitlog
            + BeeType*DurationOfVisitlog, data=data)
summary(lmfit)
```

```
##
## Call:
## lm(formula = PollenRemovedlogit ~ BeeType + DurationOfVisitlog +
##     BeeType * DurationOfVisitlog, data = data)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -1.3803 -0.3699  0.0307  0.4552  1.1611
##
## Coefficients:
##                                    Estimate Std. Error t value Pr(>|t|)
## (Intercept)                         -3.0390     0.5115  -5.941 4.45e-07 ***
## BeeTypeWorker                        1.3770     0.8722   1.579    0.122
## DurationOfVisitlog                   1.0121     0.1902   5.321 3.52e-06 ***
## BeeTypeWorker:DurationOfVisitlog  -0.2709     0.2817  -0.962    0.342
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.6525 on 43 degrees of freedom
## Multiple R-squared:  0.6151, Adjusted R-squared:  0.5882
## F-statistic:  22.9 on 3 and 43 DF,  p-value: 5.151e-09
```

```r
r <- residuals(lmfit)
yh <- predict(lmfit)

p1<-ggplot(lmfit, aes(.fitted, .resid))+geom_point()
p1 <- p1 +geom_hline(yintercept=0)+geom_smooth() +
geom_text(aes(label=ifelse((.resid>4*IQR(.resid)|.fitted>4*IQR(.fitted)),paste('', "\n", .fitted, ",",
p1
```

```
## geom_smooth: method="auto" and size of largest group is <1000, so using loess. Use 'method = x' to c
```



From the residual plot, there seem to be no outliers(see outlier detection part in the last code chunk wheere a outlier is defined if it is greater than 4*IQR(x)).

Also the p-value of cross interaction term $BeeTypeWorker : DurationofVisitlog$ is 0.342 and hence at a significance level of 0.05 can be safely neglected.

Chapter 11: 21

$$SS(\beta_0, \beta_1 \dots \beta_n) = \sum_{i=1}^{N} w_i(Y_i - \beta_0 - \beta_1 X_{1i} - \beta_2 X_{2i} - \dots - \beta_p X_{pi})^2$$

$$\frac{\partial SS}{\partial \beta_0} = 2\sum_{i=1}^{N} w_i(Y_i - \beta_0 - \beta_1 X_{1i} - \beta_2 X_{2i} - \dots - \beta_p X_{pi}) \times -1 = 0$$

$$n\beta_0 \sum w_i + \beta_1 \sum w_i X_{1i} + \beta_2 \sum w_i X_{2i} + \dots + \beta_p \sum w_i X_{pi} = \sum_{i=1}^{N} w_i Y_i$$

$$\frac{\partial SS}{\partial \beta_1} = 2\sum_{i=1}^{N} w_i(Y_i - \beta_0 - \beta_1 X_{1i} - \beta_2 X_{2i} - \dots - \beta_p X_{pi}) \times -X_{1i} = 0$$

$$\beta_0 \sum w_i X_{1i} + \beta_1 \sum w_i X_{1i}^2 + \beta_2 \sum w_i X_{2i} X_{1i} + \dots + \beta_p \sum w_i X_{pi} X_{1i} = \sum_{i=1}^{N} w_i X_{1i} Y_i$$

Similarly,

$$\frac{\partial SS}{\partial \beta_p} = 2 \sum_{i=1}^{N} w_i (Y_i - \beta_0 - \beta_1 X_{1i} - \beta_2 X_{2i} - \dots - \beta_p X_{pi}) \times -X_{1i} = 0$$

$$\beta_0 \sum w_i X_{pi} + \beta_1 \sum w_i X_{1i} X_{pi} + \beta_2 \sum w_i X_{2i} X_{pi} + \dots + \beta_p \sum w_i X_{pi}^2 = \sum_{i=1}^{N} w_i X_{pi} Y_i$$

To prove that this is indeed the minimum, we need to show that

$$\frac{\partial^2 SS}{\partial \beta_i^2}$$

is convex:

$$\frac{\partial^2 SS}{\partial \beta_0^2} = 2w_i \geq 0$$

$$\frac{\partial^2 SS}{\partial \beta_1^2} = 2 \sum_i w_i X_{1i}^2 \geq 0$$

Similarly for any $1 \leq j \leq p$:

$$\frac{\partial^2 SS}{\partial \beta_j^2} = 2 \sum_i w_i X_{ji}^2 \geq 0$$

And for

$$k \neq j$$

:

$$\frac{\partial^2 SS}{\partial \beta_j \beta_k} = 2 \sum_i w_i X_{ji} X_{ki} \geq 0$$

$$\begin{pmatrix} \sum_i w_i X_{1i}^2 & \sum_i w_i X_{1i} X_{2i} & \dots & \sum_i w_i X_{1i} X_{ni} \\ \sum_i w_i X_{2i} X_{1i} & \sum_i w_i X_{2i}^2 & \dots & \sum_i w_i X_{2i} X_{ni} \\ \vdots \sum_i w_i X_{ni} X_{1i} & \sum_i w_i X_{2i}^2 & \dots & \sum_i w_i X_{ni}^2 \end{pmatrix}$$

(We can take the w_i out by factoring that as a separate vector) and then each element in the remaining matrix in this case can be written as $H_{ij} = \mathbf{x_i}^T \mathbf{x_j}$ and hence this is a Gram matrix and positive definite, hence minima at the above point is guaranteed.