EE512 Stochastic Processes
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Independent RV $P(X = x, Y = y) = P(X = x)P(Y = y)$i or $P(X \in A, Y \in B) = P(X \in A)P(Y \in B)$

$E[X] = \sum_x xP(X = x)$ or $E[X] = \int xf(x)dx$

$Variance = E[(X - \mu)^2] = E[X^2] - (E[X])^2$

$Cov(X, Y) = E[(X - \mu_X)(Y - \mu_Y)] = E[XY] - E[X]E[Y]$

Independence $\implies$ Cov = 0

Cov =0 does not $\implies$ Indepennce. Example: $X = \{-1, 0, 1\}$ and $Y = X^2$

Indicator variable: $E[I_x] = P(I_x = 1)$

Moment generating function: $\psi(t) = E[e^{tX}]$

$\frac{d\psi(t)}{dt} = \frac{d}{dt} \int e^{tx} f(x)dx = \int xe^{tx} f(x)dx$ Thus, $\psi'(0) = E[X]$

$\psi''(t) = E[X^2]$

$\psi^n(t) = E[X^n]$

Example: Two gaussian Rv. X, Y, $Z = X + Y$ $M_Z = E[e^{tX+tY}] = E[e^{tX}]E[e^{tY}] = \psi_X(t)\psi_Y(t) = e^{\mu_x t + \frac{1}{2}\sigma_x^2} \times e^{\mu_y t + \frac{1}{2}\sigma_y^2} = e^{\mu_x t + \mu_y t + \frac{1}{2}(\sigma_x^2 + \sigma_y^2)}$

Joint MGF: $\psi(t_1, t_2, \ldots, t_n) = E[e^{\sum_i t_i X_i}]$

2.1 Conditional distrbituion and conditional expectation

$E[X_1 + X_2|Y = y] = E[X_1|Y = y] + E[X_2|Y = y]$

2.2 Conditional expectation as r.v

Y - discrete r.v.

X - disceрte r.v

$E[X|Y = y_1] = \sum xP(X|Y = y_1)$

$h(y_i) = E[X|Y = y_i]$ so h(Y) is like a r.v.

$E[h(y)] = \sum_i h(y_i)P(Y = y_i) = \sum E[X|Y = y_i]P(Y = y_i) = \sum_i \sum_j x_j P(X = x_j|Y = y_i)P(Y = y_i) = \sum_i \sum_j x_j P(X = x_j, Y = y_j) = \sum_j \sum_i x_j P(X = x_j)$

Smoothing property: $E[E[X|Y]] = EX$

$E[E[X|Y]] = \int E[X|Y = y]f(y)dy$

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$P(Z = i) = p_i$ $E[D|Z = i] = d_i \implies E[D] = E[E[D|Z]] = \sum p_i d_i$

2.3 Probability inequalities

Markov's inequality: X is non negative RV $X \geq 0$ then for $a > 0$, $P(X \geq a) \leq E[X]/a$

Proof. $E[X] = \int_0^\infty xf(x)dx = \int_0^a xf(x)dx + \int_a^\infty xf(x)dx \geq \int_a^\infty xf(x)dx \geq aP(X \geq a)$ Alter. $X \geq aI_A$

Chebychev's Inequality: X is r.v. with mean μ and $var = \sigma^2$

$P(|X - \mu| \geq a) \leq \sigma^2 a^2$

Proof: $Y = (X - \mu)^2$ $P(Y \geq a^2) \leq EY/a^2$

Jensen's Ineuqlity:

Convex function: $f(\lambda x + (1 - \lambda)y) \leq \lambda f(x) + (1 - \lambda)f(y)$

If $f(x)$ is convex then $E[f(x)] \geq f(E[X])$