1 Review

Change of Bases:

$$\mathcal{B}\{b_1, b_2, \dots b_n\} \longrightarrow B\{b_1|b_2\dots|b_n\}$$

$$\mathcal{C}\{c_1, c_2, \dots c_n\} \longrightarrow C\{c_1|c_2\dots|c_n\}$$

Diagnolization:

$$A \in C^{n\times n}$$

A is diagnolizable if $\exists\ X \in C^{n\times n}$ duch that $det(X) \neq 0$
$v \in C^{n\times n}$
$[v] = B[v]_B$
$[v]_B = B^{-1}[v]$
$[v]_C = C^{-1}B[v]_B$
$A = X\Lambda X^{-1}$, $\Lambda = diag(\lambda_1, \lambda_2, \dots \lambda_n)$ $\Lambda = X^{-1}AX$
Diagnolization is like change of basis

$$A[v] = X\Lambda X^{-1}[v]/[v]_X = [\Lambda v]_X/[\Lambda v]$$

Claim: If spectrum of A, $card(\sigma(A)) = ni.e.(\lambda_i \neq \lambda_j, i \neq j)$ then A is diagonizable(non-defective)
Proof: Do induction on # of eigen vector
$\lambda_1 \neq \lambda_2$ $c_1\vec{X}_1 + c_2\vec{X}_2 = 0$; $c_1, c_2 \neq 0$
$c_1A\vec{X}_1 + c_2A\vec{X}_2 = 0$
$c_1\lambda_1\vec{X}_1 + c_2\lambda_2\vec{X}_2 = 0$
$-c_1\lambda_2\vec{X}_1 + c_2\lambda_2\vec{X}_2 = 0$
$c_1(\lambda_1 - \lambda_2)\vec{X}_1 = 0 -> $ Contradiction
Assume truth for $k = 1$
$\sum_{i=1}^{k} c_i\vec{X}_i = 0$ $c_i isnotzero$
In particular, at least one of c_i is not zero

$$\sum_i c_i\lambda_i X_i = 0$$

$$\begin{aligned} 0 &= \lambda_k 0 - 0 \\ &= \lambda_k \sum c_i X_o - \sum_i c_i\lambda_i X_i \\ &= \sum_i c_i(\lambda_k - \lambda_i)X_i \end{aligned}$$

$$contradiction$$

X_i are independent, hence follows.

2 Interpretation

Assume $A \in C^{n\times n}$ and $\vec{b} \in C^n$; $det(A) \neq 0$
$AX = \vec{b}$
Best case:

- A is diagnolizable
- A is triangular(upper/lower)

Properties:

- Production of 2 upper triangle is upper triangular
- Inverse of non singular upper triangular is upper triangular

Argument of (2):

$$SX = I$$
$$[SX_1, SX_2, \ldots, SX_n] = [e_1, e_2, \ldots e_n](Std basis)$$
$$S\vec{X}_i = \vec{e}_i$$

Elementary row operations

- R1 Multiply on RHS by a constant
- R2 Exchanging two rows
- R3 add non zero multiple of one to another

(R2)Permutation matrix: exactly one 1 in row or column -¿ Not lower or upper traingular. But holds for R1,R3

LU decomposition $[L_1, L_2, L3]A = U$ $\hat{L}A = U$ $A = LU$ $L = \hat{L}^{-1}$

$AX = b- > Ly = b- > UX = y$

Two factorization: 1. $A = X\Lambda X^{-1}$ when X is non-defective 2. $A = LU$, when A is square and not of permutation type.

3 Norms

Absolute value:

$$|a| \geq 0$$
$$|a| = 0- > a = 0,$$
$$|ab| = |a||b|$$
$$|x + y| \leq |x| + |y|$$

Norm:

- Norm is mapping $||.|| : V- > R$ $V over C$
- ——0——: V -¿R
- $||\vec{v}|| \geq 0, ||v|| = 0$ iff $v = 0$
- $||c\vec{v}|| = |c|||\vec{v}||$
- $||\vec{v} + \vec{w}|| \leq ||\vec{v}|| + ||\vec{w}||$

p norm: $p \leq \infty$

$||\vec{v}||_p = \sum(|v_i|^p)^{1/p}$

$||v||_\infty = max|V_i|$ $1 \leq i \leq n$

Clasim: $||v||_p$ is a norm