EE596 Wavelets

Homework 1

Issued: 1/22/18

Due: 1/31/18

Problem 1. (10 pts.) Define the following time series (for $k \geq 1$):

$$x(1) = 1 \quad \text{and} \quad x(k+1) = x(k) + 1, \quad k \geq 1.$$

Use this to define a set, $\mathcal{S}$, of vectors in R^2 indexed by a non-negative integer k:

$$\mathcal{S} = \{\mathbf{x}_k\}_{k \in Z, k \geq 1}.$$

where

$$\mathbf{x}_k = [x(k) \ x(k+1)]^t, \quad k \geq 1, \quad \text{and}$$
$$\mathbf{x}_0 = [-1, 1]^t,$$

with ^{t} denoting the transpose.

a. (5 pts) Prove that for all $k_1, k_2 \geq 0$, $k_1 \neq k_2$ we have that $\mathbf{x}_{k_1}, \mathbf{x}_{k_2}$ form a basis for R^2.

b. (5 pts) Design an orthogonal basis for R^2, taking as a starting point a pair of vectors $\mathbf{x}_0, \mathbf{x}_k$, with $k \neq 0$.

Problem 2. (10 pts.) Problem 2.3, Vetterli & Kovacevic.

Problem 3. (10 pts.) Problem 2.5, Vetterli & Kovacevic.

Problem 4. (20 pts.) Problem 2.9, Vetterli & Kovacevic.

MATLAB Exercise: Analyzing Time-Frequency Properties of Orthogonal Bases (50 pts.)

In this problem we study the time-frequency properties of orthogonal bases. In particular, we are interested in the localization in time and frequency that a given basis can achieve. As discussed in class, there is a trade-off between time and frequency localization. We will examine this trade-off and the different time-frequency tilings that different bases provide by studying a signal heavily buried in noise. The signal to use is provided in the mat-file "signal_HW1.mat". Another m-file is also provided to help you get started, titled "HW1_MATLAB.m".

Two sets of bases will be studied. Code to generate each basis is provided in the m-file "GenerateBases.m". The first is a block-based DCT basis, where we apply a fixed length DCT to each segment of the signal independently. This yields a uniform time-frequency tiling, i.e., the time and frequency resolution is the same in each tile. The second basis we examine is a set of dyadic Haar basis vectors. In this case, each basis vector has a different time and frequency resolution unlike the block-based DCT basis. This leads to a very different time-frequency tiling.

First, compute and examine the N point FFT of the input signal. Discuss your observations (2 pts.). Then for each basis set, do the following (16*2 = 32 pts.):

1. (1 pt.) Check that the set of vectors generated by the "GenerateBases.m" is a basis.

2. (1 pt.) Is the basis orthogonal? Provide matlab code to verify your answer.

3. (2 pt.) Compute the projection of the input signal onto each of the basis vector.

4. (6 pts.) Generate a time-frequency plot using the projections. You can simply plot the square of each projection on the time-frequency tile of the corresponding basis vector. An example of how to do this is provided in the m-file "HW1_MATLAB.m".

5. (6 pts.) What does this time-frequency plot tell you about the signal? How is this interpretation of the signal different than when using an FFT? How is this interpretation of the signal different than when using the other basis?

Now construct your own basis. Repeat the same steps that you did for the block-based DCT and Haar bases (16 pts.). As a suggestion, you could simply change the block size of the block-based DCT. You could also use a block-based Haar basis. Or, you could construct an entirely arbitrary basis.