Problem 1 (30pts).

Define a set of signals $\mathcal{S} = \{\phi_k(n)\}_{k\in Z}$ in $l_2(Z)$. Define a new set $\mathcal{S}' = \{\phi'_k(n)\}_{k\in Z}$ in $l_2(Z)$ as follows, $\forall k \in Z$:

$$\phi'_{2k}(n) = c \cdot (\phi_{2k}(n) + \phi_{2k+1}(n)),$$

$$\phi'_{2k+1}(n) = c \cdot (\phi_{2k}(n) - \phi_{2k+1}(n)),$$

and let $x(n)$ be a signal in $l_2(Z)$ for which there exist α_k such that:

$$x(n) = \sum_{k\in Z} \alpha_k \phi_k(n).$$

1. (10 pts) As a function of the α_k, find β_k such that $x(n) = \sum_{k\in Z} \beta_k \phi'_k(n)$.

2. (10 pts) Prove that if $\mathcal{S}$ is a basis, then $\mathcal{S}'$ is also a basis.

3.(10 pts) Prove that if $\mathcal{S}$ is an orthonormal basis there is at least a c for which $\mathcal{S}'$ is also orthonormal.