

# **EE-588: Homework # 5**

Due on Wednesday, November 20, 2019

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**Problem 1**

$$y_i = \phi(a_i^T x + v_i)$$

$$a \leq \phi'(u) \leq b$$

$$v_i \sim \mathcal{N}(0, \sigma^2)$$

Define  $w_i = a_i^T x + v_i = \phi^{-1}(y_i)$ .

Given that  $y_i$  are sorted in non-decreasing order, i.e.,  $y_1 \leq y_2 \leq \dots \leq y_m$ . The derivative  $\phi'(u)$  is given by:

$$\phi'(y_{i+1}) = \frac{y_{i+1} - y_i}{w_{i+1} - w_i}$$

Since,

$$\frac{1}{\beta} \leq \frac{1}{\phi'(u)} \leq \frac{1}{\alpha}$$

we get,

$$\frac{1}{\beta}(y_{i+1} - y_i) \leq w_{i+1} - w_i \leq \frac{1}{\alpha}(y_{i+1} - y_i)$$

The likelihood is given by:

$$v_i = w_i - a_i^T x$$

$$L(w, x) = \prod_{i=1}^m \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{-\frac{(w_i - a_i^T x)^2}{2\sigma^2}\right\}$$

$$\log L(w, x) = -\frac{1}{2\sigma^2} \sum_{i=1}^m (w_i - a_i^T x)^2 - \frac{m}{2} \log 2\pi\sigma^2$$

$-\frac{m}{2} \log 2\pi\sigma^2$  is a constant, so we maximize the following function:

$$\begin{aligned} &\text{maximize} \quad -\frac{1}{2\sigma^2} \sum_{i=1}^m (w_i - a_i^T x)^2 \\ &\text{subject to} \quad w_i = a_i^T x + v_i \end{aligned}$$

```

nonlin_meas_data;
r = zeros(1,m);
r(1) = -1;
r(2) = 1;

c = zeros(1,m-1);

```

```

c(1) = -1;
Ydiff = toeplitz(c,r);

cvx_begin
    variable x(n);
    variable w(m);
    minimize (norm(w-A*x,2));
    subject to
1/beta*Ydiff*y<=Ydiff*w;
Ydiff*w <= 1/alpha*Ydiff*y;
cvx_end

disp(x);
plot(w,y);
xlabel('w');
ylabel('y');
title('ML estimate');
saveas(gcf, 'problem1.pdf');

```

## Problem 2

Given:

$$0 \leq x(1) \leq x(2) \cdots x(N)$$

$$y(t) = \sum_{\tau=1}^k h(\tau)x(t-\tau) + v(t) \quad t = 2, \dots, N+1 \quad v(t) \sim \mathcal{N}(0,1)$$

Following problem 1, the log likelihood is given by:

$$\begin{aligned} \log L(y, x) &= -\frac{1}{2} \sum_{t=2}^{N+1} v(t)^2 \\ &= -\frac{1}{2} \sum_{t=2}^{N+1} \left( y(t) - \sum_{\tau=1}^k h(\tau)x(t-\tau) \right)^2 \end{aligned}$$

which can be expressed as COP:

$$\begin{aligned} &\text{maximize} \quad -\frac{1}{2} \sum_{t=2}^{N+1} v(t)^2 \\ &\text{subject to} \quad x(N) \geq x(N-1) \geq x(N-2) \cdots x(1) \geq 0 \end{aligned}$$

```

cvx_begin
    variable x(N);
    yhat = conv(h, x);

```

```

minimize (sum_square(yhat(1:end-3)-y))
subject to
    x(1) >=0;
    x(1:N-1) <= x(2:N);
cvx_end

t = 1:N;
plot(t, xtrue, '--', t, x, 'r');
xlabel('t');
legend('xtrue', 'xhatml');
saveas(gcf, 'problem2.pdf');

```

### Problem 3

Given the inequalities,

$$f_1(x^{(j)}) > \max\{f_2(x^{(j)}), f_3(x^{(j)})\}$$

$$f_2(y^{(j)}) > \max\{f_1(y^{(j)}), f_3(y^{(j)})\}$$

$$f_3(y^{(j)}) > \max\{f_1(z^{(j)}), f_2(z^{(j)})\}$$

are symmetric, we can relax the strict inequality:

$$f_1(x^{(j)}) \geq \max\{f_2(x^{(j)}), f_3(x^{(j)})\} + 1$$

$$f_2(y^{(j)}) \geq \max\{f_1(y^{(j)}), f_3(y^{(j)})\} + 1$$

$$f_3(y^{(j)}) \geq \max\{f_1(z^{(j)}), f_2(z^{(j)})\} + 1$$

the form of the above affine functions is as follows:

$$f_i(z) = a_i^T z - b_i \quad i = [1, 2, 3]$$

The above inequalities are shift invariant for both  $a_i$  and  $b_i$ , i.e. they would not change if any constants were to be added to  $a_i$  and/or  $b_i$  across the three coefficients. WLOG, we can hence fix,  $a_1 + a_2 + a_3 = 0$  and  $b_1 + b_2 + b_3 = 0$  to solve the convex optimization problem under these two equality constraints.

```

cvx_begin
variables a1(2) a2(2) a3(2) b1 b2 b3;
a1'*X-b1 >= max(a2'*X-b2, a3'*X-b3)+1;
a2'*Y-b2 >= max(a1'*Y-b1, a3'*Y-b3)+1;
a3'*Z-b3 >= max(a1'*Z-b1, a2'*Z-b2)+1;
a1+a2+a3 == 0;
b1+b2+b3 == 0;
cvx_end

```

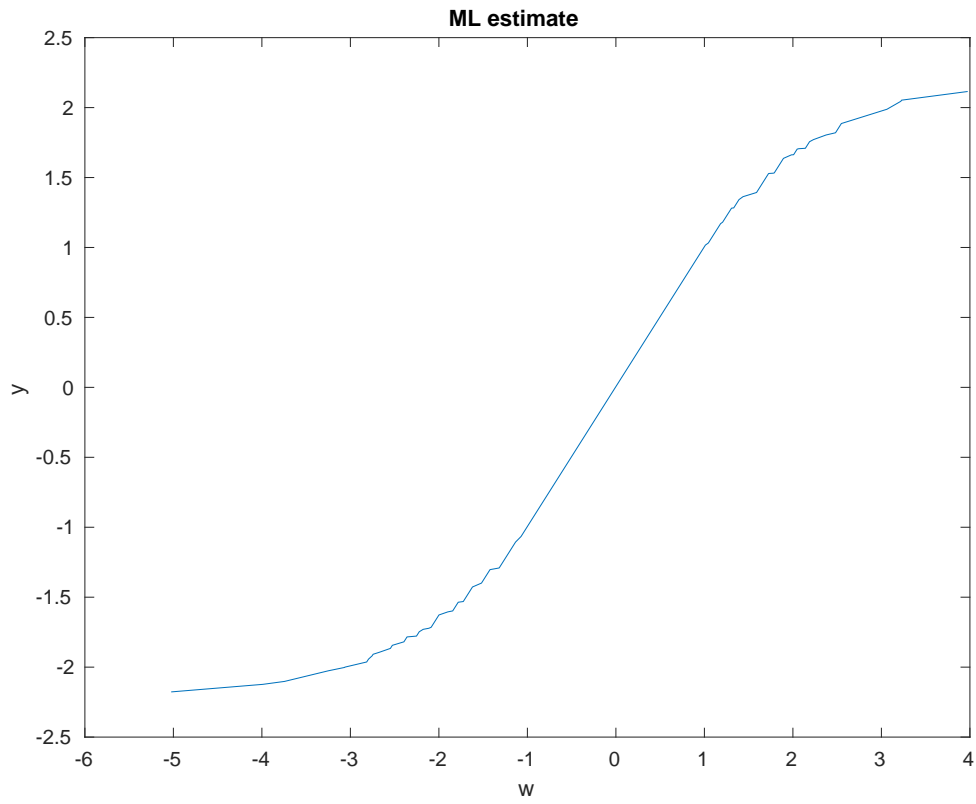


Figure 1: Problem 1

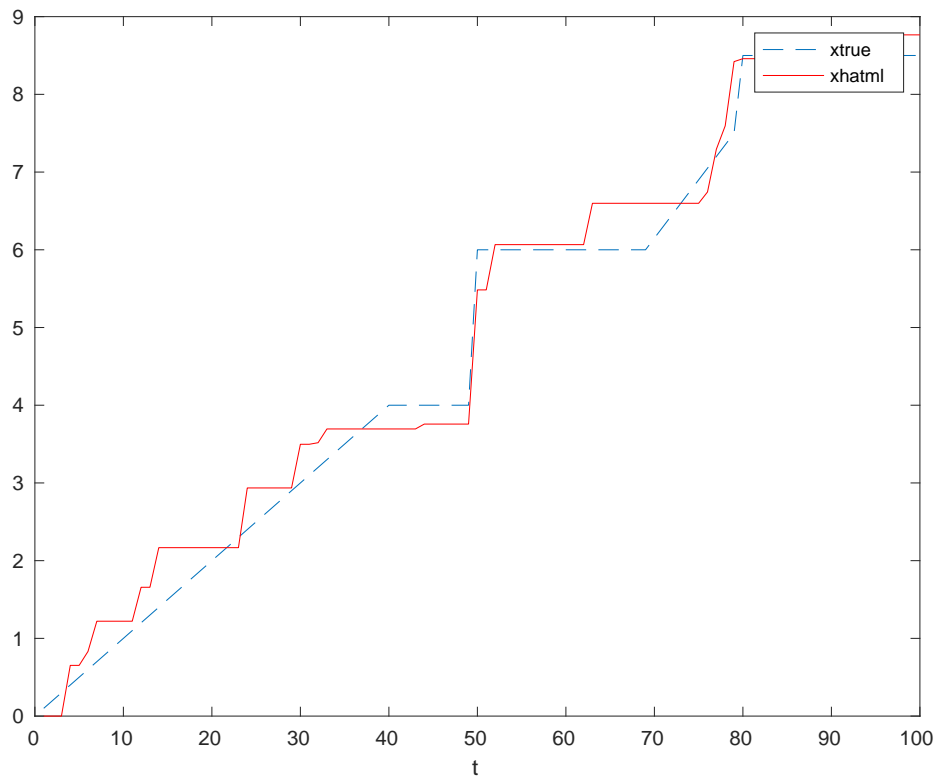


Figure 2: Problem 2

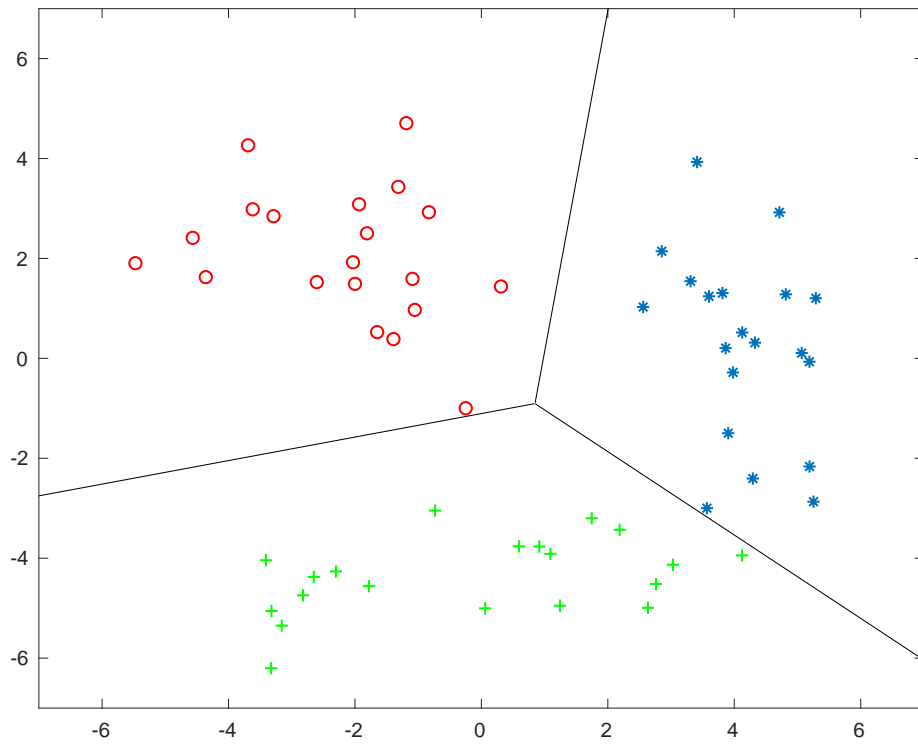


Figure 3: Problem 3