$Var(X) = 1, Var(Y) = 4, Var(Z) = 25$

Consider $var(X+Y)$ which can be written as $var(X+Y) = var(X) + var(Y) + 2cov(X,Y)$

Thus

$$var(X)+var(Y)-2\sqrt{(Var(x)Var(y)} \le var(X+Y) \le var(X)+var(Y)+2\sqrt{(Var(x)Var(y)} \tag{1}$$

$\implies$

$$1 \le Var(X+Y) \le 9 \tag{2}$$

Now consider

$$Var(X+Y+Z) = Var((X+Y)+Z) = Var(X+Y)+Var(Z)+2cov(X+Y,Z) \tag{3}$$

Fact(From Cauchy Schwartz:):

$$2cov(X+Y,Z) \ge -2\sqrt{Var(X+Y)Var(Z)} \tag{4}$$

Substituting 4 in 3:

$$Var((X+Y)+Z) \ge Var(X+Y) + Var(Z) - 2\sqrt{Var(X+Y)Var(Z)} \tag{5}$$

Let $Var(X+Y) = p$ then 5 can be rewritten as :

$$Var((X+Y)+Z) \ge p + Var(Z) - 2\sqrt{pVar(Z)} \tag{6}$$

$Var(Z)$ is a constant so we minimize $y = p - 2\sqrt{(pVar(Z))}$

$y' = 0 = 1 - \frac{\sqrt{Var(z)}}{\sqrt{(p)}} \implies p = Var(Z)$

This is indeed a minima as $y'' = \frac{1}{2}\frac{1}{p^{1.5}} > 0$

This would require p to attain a value of $p = Var(Z) = 25$ which we see from 2 is not allowed. However this y is still a decreasing function and hence we could minimize by selecting the max value of p which is 9 which gives:
$Var((X+Y)+Z) \ge 9 + 25 - 2\sqrt{3 * 5} = 4$

But the indicated answer in the class was 2.