$$
\begin{aligned}
\sum (X_i - \bar{X})^2 &= \sum (X_i - \mu + \mu - \bar{X})^2 \\
= \sum (X_i - \mu)^2 + \sum (\mu - \bar{X})^2 + 2\sum (X_i - \mu)(\mu - \bar{X}) \\
&= A + B + C
\end{aligned} \tag{1}
$$

$$
\begin{aligned}
C &= 2(\sum X_i\mu - X_i\bar{X} - \mu^2 - \mu\bar{X}) \\
&= 2(n\bar{X}\mu - n\bar{X}^2 - n\mu^2 - n\mu\bar{X}) \\
&= -2n(\mu^2 + \bar{X}^2)
\end{aligned} \tag{2}
$$

$$
B + C = n(\mu^2 + \bar{X}^2 - 2\mu\bar{X}) - 2n(\mu^2 + \bar{X}^2) = -n(\mu - \bar{X})^2 \tag{3}
$$

$$
\begin{aligned}
\sum (X_i - \bar{X})^2 = \sum (X_i - \mu)^2 - n(\mu - \bar{X})^2 \\
\frac{\sum (X_i - \bar{X})^2}{\sigma^2} = \frac{\sum (X_i - \mu)^2}{\sigma^2} - \frac{n(\mu - \bar{X})^2}{\sigma^2} \\
Q = R - T \\
R = Q + T
\end{aligned}
$$

Now, $R \sim \chi^2(n)$ and $T = \chi^2(1)$ and $M_{\chi^2(n)}(t) = \frac{1}{(1-2t)^{\frac{n}{2}}}$

Using independence of Q and T(proved in theorem's part a in class) we have $M_R = M_Q.M_T$ and so $M_Q = \frac{M_R}{M_T}$ giving: $M_Q = \frac{(1-2n)^{\frac{-n}{2}}}{(1-2n)^{\frac{-1}{2}}} = (1-2n)^{\frac{-(n-1)}{2}}$
Thus $Q = \frac{(n-1)S^2}{\sigma^2} \sim \chi^2(n-1)$